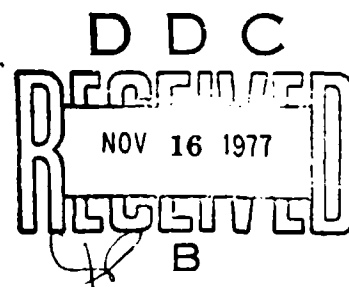


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# NAVAL POSTGRADUATE SCHOOL

Monterey, California



## THESIS

AN ANALYSIS OF THE SHIPS PARTS CONTROL  
CENTER INVENTORY MODEL AND A  
POSSIBLE ALTERNATIVE MODEL

by

Keith Wayne Lippert

June 1977

Thesis Advisor:

F. Russell Richards

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An Analysis of the Ships Parts Control Center Inventory Model  
and a  
Possible Alternative Model

by

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## ABSTRACT

The inventory model implemented by the Ships Parts Control Center (SPCC), Mechanicsburg, Pennsylvania, is a sophisticated, stationary, continuous-review, constrained reorder-level, reorder-quantity model. In accordance with DOD Instruction 4140.39, the goal of this model is "to minimize the total of variable inventory order and holding costs subject to a constraint on time-weighted, essentiality-weighted requisitions short." SPCC attempts to accomplish this goal by using various probability distributions to estimate demand during leadtime. The purpose of this thesis is to examine this existing model, to discuss the validity of its underlying assumptions when applied to a military supply system, to offer a possible alternative model using distribution free assumptions, and finally to evaluate the models using demand data obtained from the Fleet Material Support Office (FMSO), Mechanicsburg, Pennsylvania.

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## I. INTRODUCTION

The Navy Ships Parts Control Center (SPCC), located in Mechanicsburg, Pennsylvania, is the inventory control point (ICP) for approximately 459,100 items valued at \$3,714,000. These inventory items range in type from inexpensive, simple, and readily obtainable to expensive, complex, and long lead-time assemblies and components. They are used to support the active U.S. Fleet, shore activities, other military services and departments, and vessels and aircraft of approximately fifty-five foreign countries.

As in any inventory system, the paramount concern is the establishment of an inventory policy which determines when an item should be ordered and in what quantity, while ensuring a high degree of customer satisfaction. This policy must additionally operate within a given budget and within the manpower capabilities of the organization.

The inventory model presently being utilized at SPCC to meet the above requirements is a sophisticated, stationary, continuous-review, constrained reorder-level, reorder-quantity model. The decision problem is treated as decision making under risk with the distribution of leadtime demand (demands that occur between order placement and order arrival) for each item assumed known. For low-demand items, this distribution is assumed to be Poisson or negative binomial while for high-demand items, a normal distribution is used. Cumulative quarterly

demand data are kept for eight quarters, and these demands are assumed to be independent. Current procedures require that the distribution parameters (mean and variance) be updated quarterly, using the forecasting procedure of exponential smoothing to estimate mean quarterly demand ( $D_i$ ) and mean absolute deviation of quarterly demand ( $MAD_i$ ). The relationship  $\sigma_i = 1.25 MAD_i$ , which is exact for the normal distribution, is used to obtain an estimate of the standard deviation of quarterly demand.

Once these quarterly demand parameters ( $D_i$  and  $\sigma_i$ ,  $i$  indicating  $i$ th period) are specified, the leadtime demand distribution is estimated as follows:

- (1) The leadtime  $L$  is taken to be the last value experienced or a simple average of the last few values.
- (2) The mean of the distribution,  $MLD_i$ , is taken to be  $MLD_i = L \times D_i$  where  $D_i$  is the most recent forecast of mean quarterly demand.
- (3) The variance of the distribution,  $VLD_i$ , is taken to be  $VLD_i = L \times \sigma_i^2$ , where  $\sigma_i^2$  is the most recent forecast of quarterly demand variance.
- (4) The leadtime-demand distribution is then  $F(x; MLD_i, VLD_i)$ .

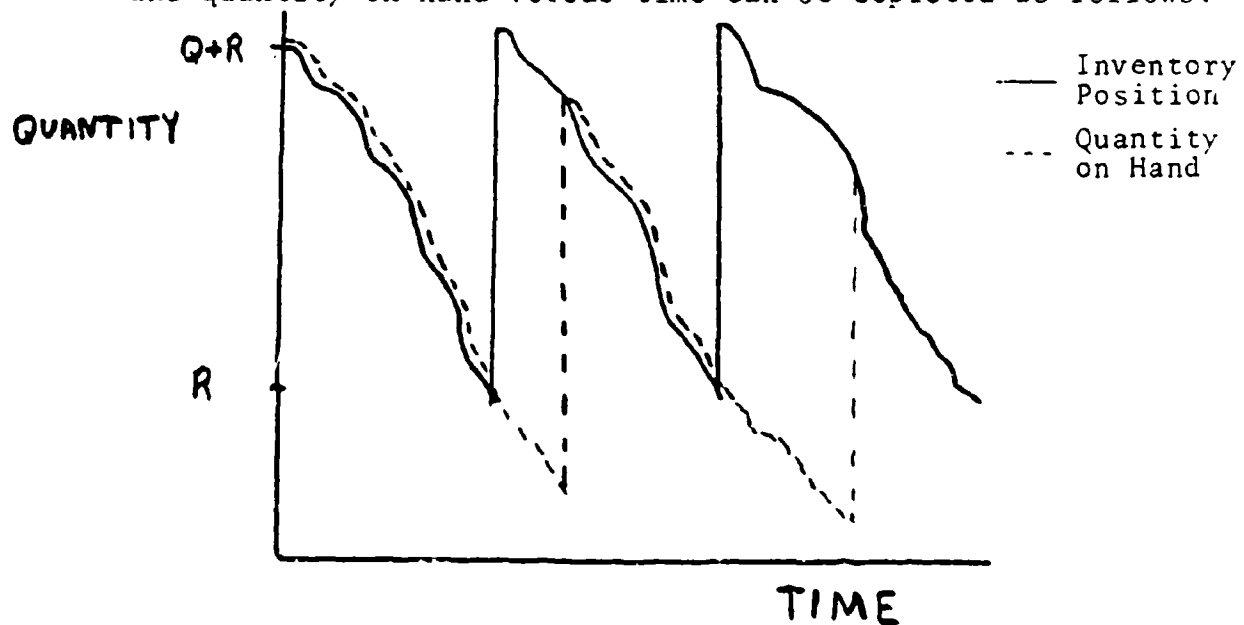
This distribution is that used in the reorder-level, reorder-quantity calculations.

The purpose of this thesis is to examine the existing inventory model which is based upon DOD Instruction 4140.39, to discuss the validity of its underlying assumptions when being applied to a military inventory system, to offer a possible alternative model utilizing distribution free assumptions, and finally, to evaluate the models using demand data

obtained from the Fleet Material Support Office (FMSO),  
Mechanicsburg, Pennsylvania.

## II. THE CURRENT INVENTORY MODEL

In order to understand the inventory model presently being used in compliance with DOD Instruction 4140.39, some definitions and assumptions are necessary. The decision variables of the inventory model are based on inventory position which is defined to be the quantity on hand plus the quantity on order minus the quantity backordered. This definition is likely to cause some confusion, but it essentially states that if there are backorders, there will be a zero quantity on hand and the inventory position is then quantity on order minus quantity backordered. In the continuous-review (Q, R) policy used, once the inventory position reaches or falls below the quantity R, Q units are ordered for stock replenishment. Hence the inventory position can exist in any "state" between R and  $Q+R$ . Graphically, inventory position and quantity on hand versus time can be depicted as follows:



As stated previously, demands occur stochastically, and initially it is assumed that demands occur according to some arbitrary distribution (F) with a quarterly mean D. Once an item is ordered, this quantity arrives in the system in a leadtime L. Demand during leadtime then is a function of leadtime and is assumed to have a mean value of MLD.

Each time an order is placed, certain setup costs (denoted here by A) are incurred. These costs can be divided into two segments, fixed and variable. The fixed segment does not depend on the frequency of reordering while the variable portion does. It is the variable segment that must be included in the cost analysis. DOD Instruction 4140.39 defines these costs to be "associated with the determination of requirements, processing of a purchase request, and subsequent contract actions through receipt of the order into the ICP system that will vary significantly in relation to the number of orders processed. Costs are considered "fixed" if they would remain constant should 50% of the work load be eliminated." Hence, A represents costs that will vary as a function of the number of times an order is placed. The fixed portion is disregarded in the calculation of A. The instruction requires that the estimate of A be updated every two years. SPCC is currently using \$70 for A.

Variable holding costs (I) are costs associated with capital, inventory losses, obsolescence, and storage. The units of I are dollars per dollar-year, i.e., inventory costs to stock \$1 worth of an item for one year. Hence, if a unit

of item  $i$  costs  $C_i$  dollars, then holding costs should equal  $I$  multiplied by  $C_i$ , multiplied by the expected quantity on hand annually. SPCC is currently using .21 for  $I$ , which consists of .1 for cost of capital, .1 for storage costs, and .01 for obsolescence costs.

Shortage costs ( $\lambda$ ) traditionally consist of two segments: shortage costs which are a function of time, and costs which are independent of time. Time dependent shortage costs in the military supply system are those costs associated with having some system out of operation or in a degraded status because of the lack of a spare part. These costs will be more severe as time passes. Shortage costs independent of time would be such things as the cost of notifying the customer that the part is not in stock and the cost of determining when the material can be supplied.

#### A. THE TOTAL VARIABLE COST EQUATION

The above definitions and assumptions are essential for understanding the total variable cost equation as defined by DOD Instruction 4140.39. This instruction states that the objective for determining procurement cycles and safety levels of supply at inventory control points for non-repairable secondary items (parts which make up principal and items) is to "minimize the total variable order and holding costs subject to a constraint on time-weighted, essentiality-weighted requisitions short." The total variable cost equation which should be minimized with respect to  $R$  and  $Q$  for an  $N$  item inventory is expressed as

$$\begin{aligned} \text{TVC} = & \sum_{i=1}^N \frac{4AD_i}{Q_i} + \sum_{i=1}^N I C_i \left( R_i + \frac{Q_i}{2} - \mu_i \right) \\ & + \lambda \sum_{i=1}^N \frac{E_i}{S_i Q_i} \int_{R_i}^{\infty} (X - R_i) [F_i(X + Q_i; L_i) - F_i(X; L_i)] dX \end{aligned}$$

where  $i$  represents the  $i$ th item.  $A$ ,  $D$ ,  $R$ ,  $Q$ ,  $\mu$ ,  $I$ ,  $C$ , and  $\lambda$  are defined as before.  $E_i$  is a weighting factor for the essentiality of the item, varying between zero and one, and  $S_i$  is the average requisition size for item  $i$ .  $F_i(X + Q_i; L_i)$  is the probability that the number of units demanded during leadtime  $L_i$  is less than or equal to  $X + Q_i$ .

The ordering cost portion of the total variable cost equation is straightforward. Intuitively, if quantity  $D_i$  of item  $i$  is the demand quantity expected during the quarter and  $Q_i$  items are ordered every time, then on the average  $\frac{D_i}{Q_i}$  orders will be placed quarterly and  $\frac{4D_i}{Q_i}$  will be placed annually. This argument can be made vigorous using renewal theory [see Ref. 6]. Multiplying this number by  $A$  and summing over all items results in  $\sum_{i=1}^N \frac{4AD_i}{Q_i}$ .

The holding cost segment is more difficult to explain. As stated previously, holding costs should be  $IC_i$  multiplied by the expected quantity on hand during the year. It is assumed that the inventory system has been operating for a "long" period and the system is in steady state. The quantity demanded by each requisition varies according to some arbitrary distribution. If the requisition interarrival times are independent and identically distributed with a finite mean, then the limiting distribution of the inventory position is uniform

on the set  $(R, R+Q)$  [see Ref. 5]. Since the inventory position (IP) is defined to be quantity on hand (OH) plus the quantity on order (OO) minus the quantity backordered (BO), then the expected quantity on hand equals  $E(IP) + E(BO) - E(OO)$ . The expected inventory position is, by definition  $1/Q \int_R^{R+Q} x dx = Q/2 + R$ . Define  $E(BO) = B(Q,R)$ , then the expected on hand quantity is  $Q/2 + R + B(Q,R) - E(OO)$ . DOD Instruction 4140.39 has chosen to disregard  $B(Q,R)$  in this expression, arguing that "this term has little effect on the optimal decision rules." However,  $B(Q,R)$  will remain in this analysis because it has a significant impact on the military supply system and is used in deriving the risk equation for SPCC's current model. If  $E(OO)$  equals  $\mu$ , then the holding cost segment of the total variable cost equation has been justified;  $\mu$  was defined as the expected leadtime and it equals  $E(OO)$  in a steady state system, as Hadley and Whitin have shown in Ref. [4]. Their argument assumed orders flow into a pipeline at rate  $D$  and procurements flow out at the same rate. The expected time an order is in the pipeline is  $L$ . Hence the expected number in the pipeline is  $DL$  which is  $\mu$  by definition.

The final segment of the total variable cost equation consists of a time-weighted, essentiality-weighted requisition's short factor. Letting  $\mu$  equal the time-weighted shortage cost, where units of  $\mu$  are dollars per time period, then  $\frac{B(Q,R)}{S}$  would be the expected number of requisitions short at any given moment. This term is multiplied by  $E$  to weight the requisitions short by the essentiality of the items. An expression for  $B(Q,R)$  must be derived.



Assume demand during leadtime  $L$  has distribution  $F(X;L)$  with density  $f(x;L)$ . Given that the inventory position is in state  $R+x$  ( $0 \leq x \leq Q$ ), the probability that  $R+x+y$  demands occur during leadtime  $L$  is  $f(y+R+x; L)$ . The probability of being in state  $R+x$  is  $1/Q$  and on integrating over all possible values of  $x$  one obtains the probability  $q(y)$  of  $y$  items being backordered:

$$\begin{aligned} q(y) &= 1/Q \int_0^Q f(y+R+x; L) dx = \\ &= 1/Q [F(y+R+Q; L) - F(y+R; L)] \quad \text{for } y \geq 0. \end{aligned}$$

The probability of being out of stock is then obtained by integrating over all possible values of  $y$ :

$$\begin{aligned} &1/Q \int_0^\infty [F(y+R+Q; L) - F(y+R; L)] dy \\ &= 1/Q \int_R^\infty [F(u+Q; L) - F(u; L)] du \end{aligned}$$

Therefore, the expected quantity backordered is

$$B(Q,R) = 1/Q \int_R^\infty (x-R) [F(x+Q; L) - F(x; L)] dx.$$

Applying the weights as previously discussed and summing over all items results in

$$\lambda \sum_{i=1}^N \frac{E_i}{S_i Q_i} \int_{R_i}^\infty (x - R_i) [F(x + Q_i; L) - F(x; L)] dx.$$

## B. THE DERIVATION OF THE RISK EQUATION AND THE REORDER QUANTITY

The procedure to minimize the total variable cost equation is to take the partial derivatives of the total variable cost

equation with respect to R and Q, set the equations equal to zero, and solve for R and Q. Dropping the summation signs for convenience, the total variable cost equation is

$$\begin{aligned} \text{TVC} &= \frac{4AD}{Q} + IC \left( \frac{Q}{2} + R - \mu + B(Q, R) \right) + \frac{\lambda E}{S} B(Q, R) \\ \frac{\partial \text{TVC}}{\partial R} &= IC + IC \frac{\partial B(Q, R)}{\partial R} + \frac{\lambda E}{S} \frac{\partial B(Q, R)}{\partial R} \\ &= IC + \frac{\partial B(Q, R)}{\partial R} \left( IC + \frac{\lambda E}{S} \right) \end{aligned}$$

Using Leibnitz's rule:

$$\begin{aligned} \frac{\partial B(Q, R)}{\partial R} &= \frac{\partial}{\partial R} \frac{1}{Q} \int_R^{\infty} (x - R) [F(x + Q; L) - F(x; L)] dx \\ &= \frac{1}{Q} \int_R^{\infty} (-1) [F(x + Q; L) - F(x; L)] dx \end{aligned}$$

Therefore,

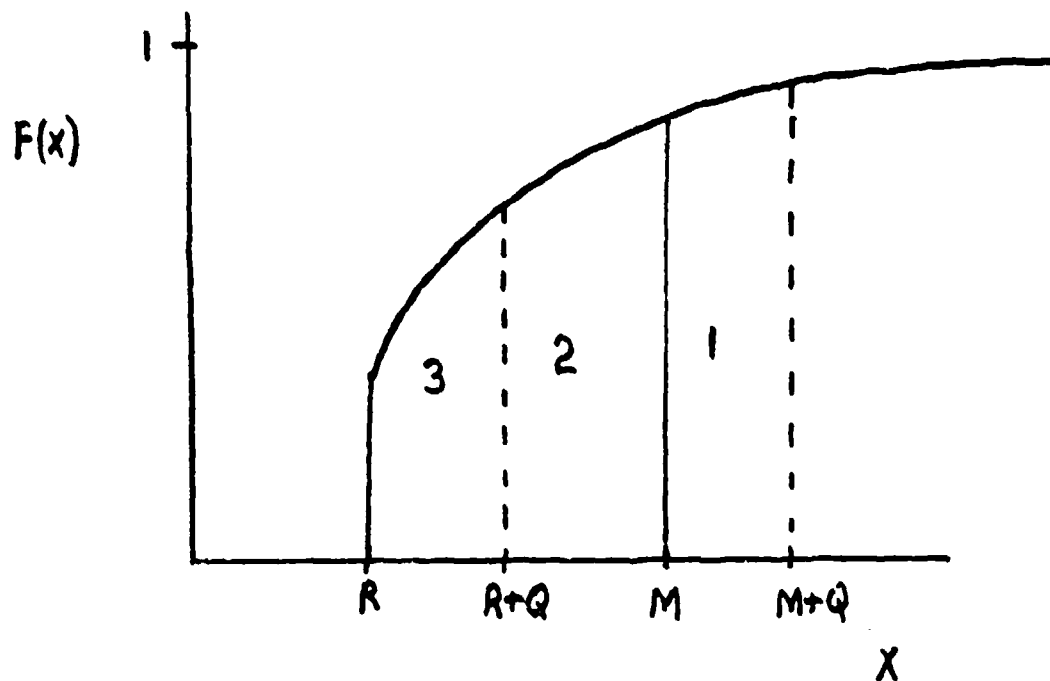
$$\frac{\partial \text{TVC}}{\partial R} = IC + \int_R^{\infty} (-1) [F(x + Q; L) - F(x; L)] dx \left( \frac{IC}{Q} + \frac{\lambda E}{SQ} \right)$$

Setting this equation equal to zero:

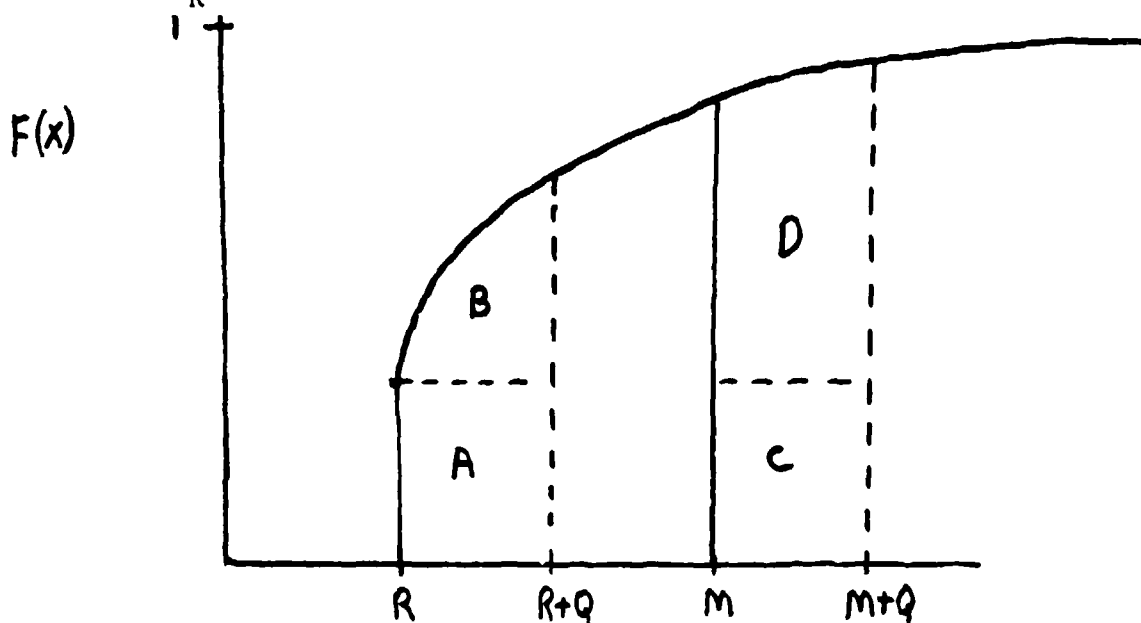
$$\int_R^{\infty} [F(x + Q; L) - F(x; L)] dx = \frac{IC}{\frac{IC}{Q} + \frac{\lambda E}{SQ}} = \frac{SQIC}{SIC + \lambda E}$$

This equation is still very difficult to solve for R since it depends on Q which is also unknown. An approximate technique to solve for R is employed. The rationale for this technique is based upon Ref. [3].

Any cumulative density function is a monotonically non-decreasing function which can be depicted as follows:



Define  $U_m(R) = \int_R^m [F(x+Q) - F(x)]dx$  which graphically is equal to area (1) minus area (3). This is equivalent to  $\int_m^{m+Q} F(x)dx - \int_R^{R+Q} F(x)dx$ . As  $m$  approaches infinity,  $F(x)$  approaches one and  $\int_m^{m+Q} F(x)dx \rightarrow Q$ . Hence  $U(R) = \lim_{m \rightarrow \infty} U_m(R) = Q - \int_R^{R+Q} F(x)dx$ . Moreover,  $a[1-F(R)] \geq Q[1-F(R+Q)]$  since  $F(x)$  is a monotonically nondecreasing function and, consequently,  $Q[1-F(R)] \geq U(R) = Q - \int_R^{R+Q} F(x)dx$ . This can be illustrated as follows:



Area (A) is equal to  $QF(R)$  which is obviously less than  $\int_R^{R+Q} F(x)dx$  which is equal to area (A) + area (B). Likewise,  $U(R) \geq Q[1 - F(R+Q)]$  since  $\int_R^{R+Q} F(x)dx \leq QF(R+Q)$ , i.e., area (C)  $\leq$  area (C) + area (D).

Therefore,  $Q(1 - F(R)) \geq \frac{SQIC}{SIC + \lambda E} \geq Q[1 - F(R+Q)]$

or,  $1 - F(R) \geq \frac{SIC}{SIC + \lambda E} \geq 1 - F(R+Q)$ .

Instead of solving for the smallest  $R$  that satisfies the above inequality,  $R$  is taken to be the solution to  $1 - F(R) = \frac{SIC}{SIC + \lambda E}$ . This expression is defined to be risk, i.e., assuming the reorder quantity  $Q$  is ordered when the inventory position is  $R$ , the risk is the possibility of being out of stock during leadtime  $L$ . Also,

$$\frac{SIC}{SIC + \lambda E} = \frac{IC}{IC + \frac{\lambda E}{S}} = \frac{DIC}{DIC + \frac{D\lambda E}{S}} = \frac{DIC}{DIC + \lambda WE}$$

where  $W$  is defined to be the quarterly requisition frequency ( $R/S$ ). This is the risk equation currently being used at SPCC.

The optimal value of  $Q$  is determined by solving  $\frac{\partial TVC}{\partial Q} = 0$ .

$$\frac{\partial TVC}{\partial Q} = -\frac{4AD}{Q^2} + \frac{IC}{2} + \frac{\partial B(Q, R)}{\partial Q} \left( IC + \frac{\lambda E}{S} \right),$$

where

$$\frac{\partial B(Q, R)}{\partial Q} = \frac{\partial}{\partial Q} \frac{1}{Q} \int_R^{\infty} (x - R) [F(x+Q; L) - F(x; L)] dx.$$

The above equation involving  $Q$  is difficult to solve explicitly. One iterative method would be to choose various values of  $R$  and  $Q$  until the equations are simultaneously solved. However, these calculations would be tedious and any gains in reduced

cost would probably be offset by the costs for the involved calculations. In actual practice, SPCC determines  $Q$  as the minimum  $[12 D_i, \text{maximum } (Q_{EOQ}, 1, D_i)]$  where  $Q_{EOQ}$  is the optimal  $Q$  for the deterministic inventory model ( $Q_{EOQ} = \sqrt{\frac{8DA}{IC}}$ ). The maximum  $(Q_{EOQ}, 1, D_i)$  ensures the reorder quantity is at least 1, which is an obvious requirement. Because of manpower restrictions, SPCC prefers to reorder an item no more than once a quarter. Hence, by ordering at least the average quarterly demand ( $D_i$ ), SPCC attempts to ensure that the total reorder workload does not exceed their reorder capacity. Finally, since the optimal  $Q$  of the DOD instruction total variable cost equation is so difficult to calculate, the  $Q_{EOQ}$  value is used as an attempt to satisfy the economic reorder quantity calculation. DOD Instruction 4140.39 additionally requires that the value of  $Q$  be no greater than three years' worth of demand.

To assure funding feasibility, the inventory system treats the shortage parameter  $\lambda$  as a Lagrange multiplier, i.e., the minimization problem can be formulated as follows:

$$\text{minimize } \sum_{i=1}^N \frac{4AD_i}{Q_i} + \sum_{i=1}^N IC_i \left( R_i + \frac{Q_i}{2} - \mu_i + B(Q, R) \right)$$

$$\text{subject to } \sum_{i=1}^N \frac{E_i}{S_i Q_i} B(Q, R) \leq F \quad (F \text{ is a function of the budget})$$

which is equivalent to

$$\begin{aligned} \text{minimize } & \sum_{i=1}^N \frac{4AD_i}{Q_i} + \sum_{i=1}^N IC_i \left( R_i + \frac{Q_i}{2} - \mu_i + B(Q, R) \right) \\ & + \lambda \left( \sum_{i=1}^N \frac{E_i}{S_i Q_i} B(Q, R) - F \right) \end{aligned}$$

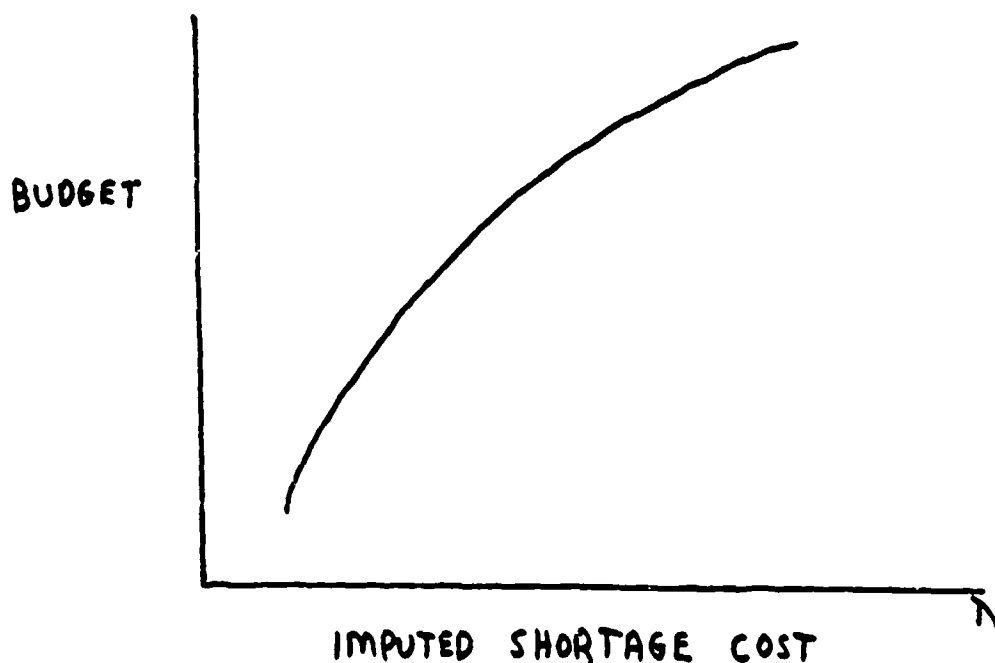
Hence, in actual practice the procedure is as follows:

- (1) Choose a value of  $\lambda$ .
- (2) Compute the reorder point  $R_i$  from the risk equation

$$1 - F(R_i) = \frac{D_i IC_i}{D_i IC_i + \lambda E_i W_i}.$$

- (3) Compute the reorder quantity  $Q_i = \min[12D_i, \max(Q_{EOQ}, 1, D_i)]$
- (4) Compute the total procurement cost during the quarter that such an  $R_i$  will produce assuming  $D_i$  units of demand.
- (5) Compare the dollar value from step (4) with funds available. Return to step (1) until funding requirements are feasible.

The relationship between  $\lambda$  and the budget can be shown to be as follows (see Ref. [7]):



Hence, if a budget is restrictive,  $\lambda$  is small, reducing R and therefore increasing the risk of being out of stock. In practice, there are too many items to determine a feasible  $\lambda$  in this manner. A random sample of items from within each budget category is taken for computational purposes.

The model and computational formulas which result from the model have been examined. The questions that remain are:

- (1) Just how close does the model approximate the real world situation?
- (2) How does the model compare to other possible models?

These questions will be discussed in the next chapters.

### III. ASSUMPTIONS AND PROBLEMS ASSOCIATED WITH THE MODEL

Clearly, the construction of any mathematical model used to represent an inventory system as complex as the military's system must employ simplifications and approximations. However, the ultimate test of a mathematical model is whether the decision rules which evolve from the model are "better" in some sense than decision rules which might be used based on best guesses or common sense. Often it is the case that the mathematical model is viewed as a panacea and is implemented even though its design and underlying assumptions do not correspond (even closely) to the real-world system. (See Ref. [10] for an overview of mathematical inventory models.) Having derived the mathematics of the inventory model used by SPCC in the previous chapter, a comparison of the model's underlying assumptions with the real-world system is examined in this chapter.

The inventory model assumes stationary demand, known item costs, and leadtimes. In many cases, these assumptions are not valid. In fact, the periodic procedure of updating the parameters (exponential smoothing used for mean demand, mean absolute deviation, and leadtimes) appears to be a tacit acceptance of the fact that the stationarity assumption is erroneous. Thus, the periodic updating of the parameters to reflect the latest demand and leadtime information represents an ad hoc adaptation of stationary inventory decision rules



to a nonstationary world. Furthermore, this stationary model was developed without any consideration given to a budget constraint or limited reordering capabilities. In describing the implementation of such a model in a real-world situation with the above restraints, Hadley and Whitin [Ref. 4, p. 403] state that it is entirely inappropriate.

"... to attempt to apply a steady state model to a situation where there (is) a fixed annual procurement budget, and even worse to do it on the basis of introducing a constraint on expected expenditures where the backorder cost was varied to bring expected expenditures in line with the budget."

Thus, the current inventory model may yield solutions that are far from optimal.

The implementation of the model additionally requires that the demand distribution and the demand distribution parameters be known. (The Poisson and negative binomial distribution are used for items with leadtime demands less than twenty. For items with leadtime demands greater than twenty, the normal distribution is used.) In many cases the item demands are so erratic that it is unreasonable to assume that any standard distribution could reasonably approximate the real-world situation. Appendix A illustrates these erratic demand patterns for a random sample of the items.

The large number of items being managed by the Navy supply system also presents numerous problems. Because of their large number, the inventory items, for some calculations, can not be treated separately. This results in generalizations that may be detrimental to the system as a whole. For example, items with the same cognizance symbol are budgeted together,

and hence, when funding is received, compete with each other for those funds. The computational formula for determining R as previously derived requires a  $\lambda$  to be chosen from a random sample of items with the same cognizance symbol. The available funding determines  $\lambda$  which in turn determines R and Q. Hence, funding feasibility determines the time weighted shortage factor  $\lambda$ . This policy implies that the shortage cost of each item within a cognizance symbol is the same; this is totally unrealistic in the real-world system. For example, the cost of being out of stock for an item which results in a delay in the deployment of a vessel is hardly comparable to being out of stock of a movie projector light bulb. Item essentiality values can partially offset this disparity. However, adequate differentiation is not provided because of the failure to use these values adequately in actual practice.

The estimation of ordering and holding costs can also present significant problems. As indicated previously, the cost to order, A, includes "costs associated with the determination of requirements, processing of a purchase request, and subsequent contract actions through receipt of the order into the ICP system." Only variable costs with respect to the number of orders placed are considered. DOD Instruction 4140.39, in fact, enumerates various items which should be included in determining ordering costs. These categories are reproduced in Appendix B.

It should be obvious from Appendix B that the accuracy of determining these costs in the calculation of A are nebulous

at best. The obvious question is, just how sensitive are the current solutions to changes in A? In simple deterministic inventory models, it can be shown that the optimal total variable cost is rather insensitive around the true value of A. In the current model, A has no effect on the reorder point since A is not present in the risk equation. However, if the reorder quantity  $Q = \min (l2D_i, \max (Q_{EOQ}, l, D_i))$  is in fact  $Q_{EOQ} = \sqrt{\frac{8DA}{IC}}$ , then A has a definite impact. A sensitivity analysis involving the effect of A on total variable cost is very cumbersome and obviously depends on the underlying distribution of leadtime demand. It is extremely difficult to get a simple structure for total variable cost as a function of A because Q appears in  $F(x+Q; L)$ . The problem is complicated further by the fact that Q is not always  $Q_{EOQ}$ . However, as in the deterministic inventory model, total variable cost should be relatively insensitive to varying values of A because of the smoothing off of the "rough edges" of the deterministic backorders model when expectations are taken.

The exact formulation of holding costs (I) is probably more difficult to determine than A. The calculation of I, as with A, disregards any cost as fixed if it would remain constant should 50% of the workload be eliminated. In profit organizations, the opportunity cost portion of I is the rate of return an organization could obtain had the funds been invested elsewhere. This figure has traditionally been around .1. For the military supply system, this is the cost of investing in inventory rather than buying other equipment

(planes, missiles, etc.). This figure is difficult to ascertain directly, but DOD Instruction 4140.39 requires the use of .1.

Obsolescence costs are also a significant factor in the holding costs for military supply systems. Accurate figures are again difficult to ascertain. As indicated previously, SPCC uses .21 for the total holding cost rate. However, inflation has increased significantly since 1963 when the .21 figure was estimated and the figure may be too low. Again, the obvious question is, just how sensitive is the model to various values of  $I$ ? As before, total variable cost is relatively insensitive to values around the true value of  $I$  for simple deterministic models. The sensitivity of the current model to the holding cost rate will be discussed later.

Total variable cost is the traditional measure of effectiveness for inventory models. For profit-oriented companies, this measure of effectiveness seems to be realistic. DOD Instruction 4140.39 modifies this measure of effectiveness by appending on a constraint of time weighted, essentiality weighted requisitions short. However, because of the aforementioned difficulties in determining  $A$  and  $I$  and the fact that  $\lambda$  is a function of the budget, it appears that this measure of effectiveness may have very little significance. When viewed within the context of a military supply system, a more reasonable measure of effectiveness may be percentage of requisitions immediately filled without backordering. From the operating forces viewpoint, this figure has the most significance.

Currently, the risk of an item's being out of stock is a function of the budget while the reorder quantity is independent of this constraint. A more reasonable approach may be to determine the risk of an item's being out of stock during leadtime independently of the budget. The risk figure could be determined on the basis of essentiality and demand, i.e., the higher the essentiality the lower the risk, and likewise, the higher the demand the lower the risk. The reorder quantity should then be dependent on the budget. The proposed alternative in the next chapter will incorporate these ideas.

Finally, the assumption of a known leadtime demand distribution may be the most difficult to justify of all. Demand patterns for many items are extremely erratic and difficult to predict. Appendix A illustrates these erratic demand patterns. Standard distributions fit this type of data very poorly. (See Ref. [8] which attempted to fit a normal distribution to samples of military demand data. Statistical tests rejected the normal distribution in all cases.)

As discussed above, it appears that the current model and procedures for various reasons deviate from the real-world situation by a considerable degree. It is the purpose of the next chapter to offer an alternative to the above model.

#### IV. AN ALTERNATIVE PROCEDURE

As indicated in the previous chapter, the underlying assumptions and procedures of the current model may deviate from the real-world system to such an extent that simple alternative models may be more effective. Based on the criticisms of the previous chapter, any alternative model should try to avoid or at least minimize the current model's weaknesses.

The alternative presented here will utilize a nonparametric approach based on order statistics for the decision variable. (See Ref. [1] and [9] for discussions on order statistics.) The objective of the model is to improve the requisition effectiveness of SPCC's current model while still operating within a given budget. This is accomplished by setting the risk at a lower level (.1 for the deviation to follow) and determining the reorder quantity as a function of the budget. In mathematical terms, the objective of the alternative model is for all items  $i$ ,  $P(x_i \geq R_i) \leq \rho_i$  for fixed  $\rho_i$  when  $0 \leq \rho_i \leq 1$  and operate in such a way that the total procurement budget is less than or equal to a given budget. This policy is implemented without any assumptions about costs (other than procurement) and distributions.

##### A. THE DEVIATION OF THE RISK EQUATION

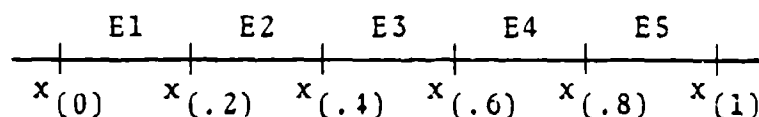
Assume  $n$  periods of demand for an item are observed. Arrange these observations from the smallest to the largest,

i.e.,  $x_{(1)} \leq x_{(2)} \leq x_{(3)} \leq \dots \leq x_{(n)}$ . Designate the next observation  $x^*$  which can assume any one of  $n+1$  positions, i.e., each position before the already observed values or one greater than  $x_{(n)}$ . Hence, the probability of  $x^*$  being in any one of these  $n+1$  positions is  $1/n+1$ . For any given value, call it  $x_{(k)}$ , the probability that  $x^* > x_{(k)}$  is  $\frac{n+1-k}{n}$ .

Now assume that the protection level is 0.9 (the risk of being out of stock during leadtime is equal to 0.1) and the leadtime is equal to one period. Then  $\frac{n+1-k}{n} = 0.1$ , and solving for  $k$  results in  $k = 0.9n + 1$ . Hence, use the  $0.9n + 1$  order statistic as the reorder point.

For leadtimes greater than one period, the analysis is more complicated. Let  $x_{(.2)}$  denote the 20th percentile and divide the underlying demand distribution into the following percentiles:

Figure 1



E1 represents the first interval, E2 represents the second interval, etc. Now assume the leadtime is two periods. The problem is to find the smallest  $r$  such that  $P[x + y \leq r] \geq .9$  where  $x$  is the demand in the next period and  $y$  is the demand in the second following period. Then

$$P[x + y \leq r] = \sum_{x=0}^r P[y \leq r - x] P[X = x].$$

In each of the five intervals in Figure 1, approximate the value of the interval by its midpoint, i.e., interval E1 by  $x_{(.1)}$ , interval E2 by  $x_{(.3)}$ , etc. Hence

$$P[x + y \leq r] = P[y \leq r - x_{(.1)}]P[x \in E1] + P[y \leq r - x_{(.3)}]P[x \in E2] \\ + \dots + P[y \leq r - x_{(.9)}]P[x \in E5].$$

Set this equation equal to .9. Therefore

$$(.2)[P[y \leq r - x_{(.1)}] + P[y \leq r - x_{(.3)}] + P[y \leq r - x_{(.5)}] \\ + P[y \leq r - x_{(.7)}] + P[y \leq r - x_{(.9)}]] = .9.$$

Designate each of the five terms in the brackets  $P_1, P_2, P_3, P_4$ , and  $P_5$ . Then,  $\sum_{i=1}^5 P_i = \frac{0.9}{0.2} = 4.5$ . Also,  $P_1 \geq P_2 \geq P_3 \geq P_4 \geq P_5$ . Since the sum of the five terms must equal 4.5 and the maximum value that any one term can assume is 1, this implies that the smallest value  $P_5$  must be between .5 and 1 for the equality to be true. Hence,  $P[y \leq r - x_{(.9)}] \geq .5$  which implies that  $r$  must be at least  $x_{(.9)} + x_{(.5)}$ . Hence, choose the reorder point to be  $x_{(.9)} + x_{(.5)}$ . Similar analysis can be conducted for leadtimes of three and four periods. For leadtimes between integer value leadtimes, simple linear interpolation can determine the reorder point.

Assume that 20 observations of demands are as follows:  
0, 0, 0, 0, 0, 1, 1, 4, 4, 5, 8, 12, 15, 20, 30, 33, 37, 40,  
40, 60. Hence the following are the reorder points for varying values of leadtime.



<u>Leadtime in Quarters</u>	<u>Reorder Point</u>
1	40
1.5	44
2	47
2.2	51
3	67

#### B. THE DERIVATION OF THE REORDER QUANTITY Q

The next step is to determine the reorder quantity Q. As before, Q should be free of all cost (except procurement cost) assumptions and distribution assumptions, but it must incorporate the budget constraint to assure funding feasibility. Furthermore, Q should be a function of the rate of demand, the unit cost, and item essentiality. Intuitively, the greater the demand and the higher the essentiality, the larger Q should be, and the greater the cost of the item, the smaller Q should be. Combining these ideas with the evidence contained in mathematical expressions for the optimal order quantities in a variety of different models (See Ref. [10]) indicates that Q should be directly proportional to the square root of the product of the demand rate and the essentiality and inversely proportional to the square root of the unit cost. Finally, the reorder quantity should comply with the restriction set forth in DOD Instruction 4140.39 that Q should be a minimum of three months of demand and a maximum of three years of demand. These considerations give rise to the following algorithm for determining Q. Let  $M_1$  be the estimated median

demand rate for item  $i$ , let  $B$  be the budget available for the given period, and let  $N = \{i : IP_i - M_i \leq R_i\}$  where  $IP_i$  is the inventory position for item  $i$ . Then

$$(1) \text{ Define } Q_i = k \sqrt{\frac{M_i E_i}{C_i}}$$

$$(2) \sum_{i=1}^N C_i Q_i = k \sum_{i=1}^N \sqrt{C_i M_i E_i} = B \Rightarrow k = \frac{B}{\sum_{i=1}^N \sqrt{C_i M_i E_i}}$$

(3) Evaluate  $Q_i$ , if  $Q_i \geq M_i$  for all  $i$ , then end; if not go to 4.

(4) Define  $N^1 = \{i : Q_i < M_i\}$ , for items belonging to  $N^1$ , define  $Q_i = M_i$ .

$$(5) \text{ Define } B^1 = B - \sum_{i=1}^{N^1} C_i Q_i, \text{ recalculate } k = \frac{B^1}{\sum_{i=1}^{N-N^1} \sqrt{C_i M_i E_i}}$$

Return to step (3) until  $Q_i \geq M_i$  for all  $i$ .

An example may be illustrative at this time. Suppose there is a three item inventory system such that all three items are elements of  $N$ . Further, let

$$B = \$700$$

$$C_1 = \$10 \quad E_1 = 1 \quad M_1 = 5$$

$$C_2 = \$20 \quad E_2 = .8 \quad M_2 = 3$$

$$C_3 = \$100 \quad E_3 = 1 \quad M_3 = 5$$

Then,  $k = \frac{700}{36.36} = 19.25$ . Hence  $Q_1 = 14$ ,  $Q_2 = 7$ ,  $Q_3 = 4$ .

$Q_3$  must be increased to 5. Then  $B^1 = 700 - 5(100) = 200$ .

$k$  now equals  $\frac{200}{13.99} = 14.29$ , then  $Q_1 = 10$  and  $Q_2 = 5$ . Since all  $Q_i \geq M_i$ , the algorithm is ended.

Current procedures require that demand data be maintained on a quarterly basis. However, were this alternative procedure adopted, maintaining demand data on a monthly basis would allow much more flexibility. If adopted, a quarterly supply demand review could be conducted to update the order statistics. The three oldest observations would be eliminated with the three newest observations replacing them. The reorder quantity (Q) would be updated at the quarterly review.

The next chapter will analyze the two models using the data provided by FMSO.

## V. EVALUATION OF THE MODELS

As mentioned previously, demand data for five thousand items was obtained from the Fleet Material Support Office, Mechanicsburg, Pennsylvania. The following information was extracted from the computer tape and used in the evaluation: cost, leadtime, mean absolute deviation of quarterly demands, quantity on hand, quantity on order, and the demand quantities for eight quarters. The simulation models used to evaluate the inventory procedures and the assumptions associated with each simulation are discussed in this chapter.

### A. BACKGROUND

The historical data contained eight observations for each item. The data represented the cumulative demand for each item during the previous eight quarters and did not delineate how many requisitions made up this demand or when during the quarter the demand occurred. In order to use this data, it was assumed that only one requisition occurred during each quarter and that the single requisition was for the entire quantity. A random number was drawn to determine when during the quarter the demand occurred. For comparison of models, the same random numbers were used in the simulation.

To avoid excessive computer time, a random sample of 500 items from the 5,000 received was utilized in this analysis. Without any knowledge of an appropriate budget figure for these items, an arbitrary value for the shortage parameter

$\lambda$  was chosen to be \$100 per year. This value resulted in a low theoretical risk and a large budget. The dollar value which resulted from this choice of  $\lambda$  was then used as the budget figure in the alternative model. For simplicity, a risk setting of .1 was established to be used in the alternative model. This value was used for all items, while in actual practice this number could vary depending on the essentiality of the item.

An examination of the data revealed that the leadtime MAD numbers were extremely small and not consistent with the demand data. Using these small numbers (most items had a MAD of .1 quarters) would have been entirely unrealistic. Discussions with SPCC revealed that the MAD numbers had been forced to these small figures because the other calculations of MAD (exponential smoothing) had yielded very large estimates of variance and a decision had been made not to use these large values. However, a regression analysis had been conducted at SPCC relating leadtime demand to leadtime MAD. This equation is used to estimate MAD when exponential smoothing gives unreasonable values. As in actual practice, only items having quarterly demand greater than five units had MAD values updated using exponential smoothing while the remaining items used the regression equation.

As previously mentioned, SPCC uses the normal, negative binomial, and Poisson distributions to approximate leadtime demand. For this analysis, the negative binomial distribution was used for items with leadtime demand less than twenty units

and the normal distribution was used for items with leadtime demand greater than or equal to twenty units.

Many of the items had an on hand quantity of zero and none on reorder. This may have been caused by low demand or a restrictive budget. Once a demand for these items occurred, not only was this demand backordered but additional units had to be placed on order so that the inventory position was between the calculated  $R$  and  $Q+R$  as required. When this condition occurred, only one ordering charge (\$70) was added in calculating the total variable cost. The holding cost was set equal to .21 and the order quantity was restricted to be no more than three years' worth of demand.

The methodology employed in the simulation of the current model examined the eight quarters of demand for each item in turn, determined the cost to operate the system ( $C$  multiplied by  $Q$ ), determined the value of the safety stock ( $C$  multiplied by  $R$ ) and determined the percent of requests that were immediately filled. In addition to evaluating these values for the 500 items, grand totals were also computed. During the simulation, the actual total variable cost was calculated. This value was updated quarterly by adding \$70 for  $A$  each time an item was ordered. The quarterly average on hand quantity was calculated and multiplied by the value of  $IC$  to determine holding costs. Finally, the quantity backordered was multiplied by \$100 to determine shortage costs. Technically, the backorder term should have been multiplied by  $\lambda$  (\$100) divided by the average requisition size. Since this number was not

readily available, the total variable cost value is in error by a factor, but the factor is the same for each model. The program and sample output for the current model is exhibited in Appendix C.

The methodology employed for the alternative model was somewhat different. The demand observations were arranged into order statistics and the reorder point determined as discussed in the previous chapter. The reorder quantity was determined as a function of the budget. The budget used was the dollar value ( $\sum C_i Q_i$ ) which was necessary to operate the current model's simulation with  $\lambda$  equal to \$100. The simulation examined all items for quarter one, the budget was updated, quarter two was examined, etc. The same output as the current model was printed out. The program and sample output for the alternative model is exhibited in Appendix D.

Iterations of the program simulating the current model were run to determine the frequency of each of the reorder quantity options, i.e.,  $Q_{EOQ}$ , I, D, or three years' worth of demand. Additionally, the values of A and I were allowed to vary to determine the sensitivity of the model to these parameters. Other iterations involved forcing the use of  $Q_{EOQ}$  as the reorder quantity. The alternative model was run with the restriction that the reorder quantity could only be as large as three years' worth of demand.

Finally, the DOD Instruction 4140.39 total variable cost equation was used to compare what the equation would predict versus what the models actually produced. This was accomplished

as follows. The quarterly demand was multiplied by the lead-time to estimate demand during leadtime. MAD was evaluated by exponential smoothing or the previously mentioned regression equation. The values were updated quarterly in addition to updating R and A with risk set at .1. The expected quantity backordered ( $\frac{1}{Q} \int_R^{\infty} (x-R) [F(x+Q;L) - F(x;L)] dx$ ) caused difficulties because no known closed expression for the cumulative distribution function for the negative binomial is available. However, an expression for the normal distribution is available.

Let  $I(R) = \int_R^{\infty} \phi(x) dx$  where  $\phi(R) = \frac{1}{\sqrt{2\pi}} e^{-\frac{R^2}{2}}$  is the density function for the standard normal distribution,  $N(0,1)$ . Then

$$\begin{aligned} \frac{1}{Q} \int_R^{\infty} (x-R) [F(x+Q;L) - F(x;L)] dx &= \frac{1}{Q} \int_0^{\infty} y [F(y+R+Q;L) \\ &- F(y+R;L)] dy \\ &= \frac{1}{Q} \int_0^{\infty} y [\bar{F}(y+R;L) - \bar{F}(y+R+Q;L)] dy \end{aligned}$$

By subtracting the mean of leadtime demand and dividing by the standard deviation of leadtime demand to standardize the random variable, the above expression is equivalent to

$$\frac{1}{Q} \int_0^{\infty} y [\phi(\frac{y+R-\mu}{\sigma}) - \phi(\frac{y+R+Q-\mu}{\sigma})] dy$$

Using two equations from Ref. [4], namely

$$\int_R^{\infty} \phi(x) dx = \phi(R) - R\phi(R) \quad \text{and}$$

$$\int_R^{\infty} x\phi(x) dx = \frac{1}{2} [(1-R^2)\phi(R) + R\phi(R)] ,$$

it can be shown that the above equation reduces to



$$\frac{1}{2} [\sigma^2 + (R-\mu)^2] \phi\left(\frac{R-\mu}{\sigma}\right) - \frac{\sigma}{2} (R-\mu) \phi\left(\frac{R-\mu}{\sigma}\right) \\ - \left(\frac{1}{2} [\sigma^2 + (R+Q-\mu)^2] \phi\left(\frac{R+Q-\mu}{\sigma}\right) - \frac{\sigma}{2} (R+Q-\mu) \phi\left(\frac{R+Q-\mu}{\sigma}\right)\right).$$

This expression was incorporated in the program as the expected quantity backordered for items whose leadtime demand was approximated by the normal distribution.

The results are summarized in the following tables.

TABLE I

Reorder Quantity Option

	<u>Q<sub>EOQ</sub></u>	<u>1</u>	<u>Quarterly Demand</u>	<u>3 Years Demand</u>
Current Model	25%	2%	2%	71%

TABLE II

Forced Q<sub>EOQ</sub>, Vary I, A Fixed

<u>I</u>	<u>Total Reorder Cost</u>	<u>Safety Stock Cost</u>	<u>Percent Filled</u>
.1	\$1,278,280	\$459,734.63	.4963
.21	1,232,060	444,943.38	.4911
.32	1,220,270	438,272.56	.4864
.43	1,220,248	434,858.19	.4772

TABLE III  
Forced  $Q_{EOQ}$ , Vary A, I Fixed

<u>A</u>	<u>Total Reorder Cost</u>	<u>Safety Stock Cost</u>	<u>Percent Filled</u>
35	\$1,234,250	\$444,943	.4791
70	1,232,060	444,943	.4911
105	1,242,770	444,943	.4845
140	1,262,053	444,943	.4944

TABLE IV  
Comparison of Total Variable Costs for  
Demands Approximated by Normal Distribution  
(43 items out of 500, risk = .1)

	<u>Ordering Costs</u>	<u>Holding Costs</u>	<u>Total</u>	<u>No. Items Backordered</u>
Current Model	\$2310	\$36,694	\$39,004	1910
Current Model, $Q_{EOQ}$ Forced	2310	36,439	38,749	1897
TVC Equation	6399	45,740	52,139	260

TABLE V  
Comparison of the Models for Fill Rate of 90%

	<u>Surrogate for Ordering Costs</u>	<u>Surrogate for Holding Costs</u>	<u>Safety Stock Cost</u>
Current Model	\$177.96	\$10,547,000	\$444,943
Alternate	458.48	957,830	924,275

## B. ANALYSIS

Table I indicates that of the four reorder quantity options, the overwhelming majority of the items in this sample used three years' worth of demand as the reorder quantity. Because of this fact, the current model is relatively insensitive to the values of A and I. Hence, unless the theoretically correct values of A and I are significantly different from the values of A and I currently used, little change can be anticipated.

Tables II and III forced the value of  $Q_{EOQ}$  to be the reorder quantity. By increasing I, the size of Q would decrease as would R. As anticipated, this is exactly what occurred, as indicated in Table II. The reorder cost ( $CxQ$ ) decreased while the safety stock ( $RxC$ ) also decreased. Table III indicates that varying the value of A has no impact on the cost of the safety stock. Since A does not appear in the risk equation, this is the anticipated result. The total reorder cost showed a slight decrease when A increased from 35 to 70, but increased as anticipated as A increased to 140.

Table IV compares the current model and the forced  $Q_{EOQ}$  model with what the DOD Instruction 4140.39 total variable cost equation would predict for items approximated by the normal distribution. The most interesting disparity is in the number of items backordered. The two models incurred backorders over seven times what the equation would predict using the normal distribution. This clearly demonstrates the inadequacy of the assumption of the normal distribution

in estimating leadtime demand. As stated previously, the real-world demand data is simply too erratic for any standard distribution.

Little variability in any of the measures of effectiveness for the SPCC model and the proposed alternative occurred in any of the computer simulations. This was particularly true in the comparison of percent of requisitions filled. This small variability was probably caused by the fact that many of the items either had none on hand or relatively small quantities. Any initial demand of a comparatively large size resulted in a low percentage of requisitions filled. With leadtimes of one to four quarters, the system never had an opportunity to offset this initial setback in the eight quarters of data available. Likewise, the data base did not provide ample time to differentiate among ordering costs, holding costs, and shortage costs. Eight quarters of data appeared to be just adequate to allow the systems to start operating in accordance with their respective inventory policies. Using the above mentioned measures of effectiveness, valid comparisons can only be conducted with demand data maintained for a longer time period.

However, the simulations did reveal some significant differences between the current model and the alternative. As Table V indicates, the value of the safety stock ( $RxC$ ) for the alternative model was approximately double that of the current model. With the higher reorder values and with future demands exhibiting similar quantities, it is anticipated

that the percentage of requisitions immediately filled would increase significantly using the proposed alternative.

In an attempt to try to differentiate between the models, surrogates for ordering and holding costs were calculated. Each of the models was allowed to operate for seven quarters. At the end of seven quarters, the values of R and Q for the 500 items were determined for each of the models. These values were used as input to a computer program which calculated the surrogates for ordering and holding costs. The ordering cost segment of the total variable cost equation is equal to  $\sum_{i=1}^N \frac{4AD_i}{Q_i}$  as previously indicated. Since the value of  $4AD_i$  is the same for each model,  $\sum_{i=1}^N \frac{1}{Q_i}$  was used as a surrogate. For holding cost,  $\sum_{i=1}^N C_i (R_i + \frac{Q_i}{2})$  was used as a surrogate since these values were readily available. The results of the computer program are exhibited in Table V. The values indicate that a significant reduction in holding costs can be anticipated using the alternative model. However, this decrease in holding costs would be partially offset by increased ordering costs. Comparisons of the reorder points and reorder quantities of the two models revealed that in general the reorder points were higher and the reorder quantities were lower using the alternative model. Additional data would be necessary for a final evaluation, but the surrogates indicate that the alternative model may be the better model.

## VI. SUMMARY AND CONCLUSIONS

Chapter II discussed the derivation of the total variable cost equation and the risk and reorder quantities which make up the current inventory policy. Chapter III discussed the many assumptions that are necessary in order for SPCC's current inventory model to be an "accurate" representation of the real-world system. As indicated, many of these assumptions simply are not applicable to the military supply system. The use of a steady state model with probability distributions used to estimate leadtime demands intuitively does not seem appropriate for a system that is constantly changing with time, which has a significant budget constraint, and which has a widely fluctuating demand pattern.

The possible implementation of an inventory model which can eliminate or minimize these questionable assumptions seems justified. The proposed alternative model is an attempt to do just that. The alternative model is appealing because it divorces itself from any probability distribution by using nonparametric statistics. Additionally, it is an inventory model which does not rely on long-run steady-state assumptions. The model is flexible in the sense that the risk the item manager is willing to assume is predetermined and not a function of the budget. Additionally, the reorder quantity is a function of the budget and does not require any assumptions about holding, ordering and shortage costs.

An accurate comparison of the two models is extremely difficult because of the limited data base that is available. In order to use the real-world data, many assumptions had to be used in the simulation which may have been too restrictive for a valid comparison. Ideally, the models should be evaluated with demand data available on a monthly basis. Additionally, the quantity of each requisition and when it arrived in the system are necessary for an accurate simulation.

The significant difference of the alternative model is the drastic increase of safety stock and the potential of reduced total variable costs. In order to effectively evaluate whether the increase in safety stock is justified, more than eight quarters of data are necessary. It is believed that with a large data base, the increased value of the safety stock will be the determining factor in the final evaluation of the proposed model.

# APPENDIX A

## QUARTERLY DEMANDS FOR TEN ITEMS

Item	Quarter:							
	1	2	3	4	5	6	7	8
1.	0	0	0	5	1	0	5	19
2.	0	10	0	0	0	0	0	10
3.	0	0	6	0	0	0	0	0
4.	1	0	8	0	0	0	0	20
5.	0	0	34	0	6	5	0	6
6.	1	17	0	2	0	0	0	0
7.	116	60	0	220	0	0	20	100
8.	60	0	0	0	0	0	0	100
9.	1	5	0	0	3	0	0	0
10.	6	0	0	5	6	0	0	0



## APPENDIX B

### FUNCTIONAL ELEMENTS TO BE INCLUDED IN COST TO ORDER AT THE INVENTORY CONTROL POINT (ICP) LEVEL

#### I. DIRECT LABOR/ADP COSTS PER ITEM PROCURED AT ICP (Exclusive of any contract administration function not listed)

##### A. Processing Purchase Request (PR) to Procurement

	<u>Labor</u>	<u>ADP</u>
1. Preparation of documents which recommend the buy	\$ _____	\$ _____
2. Item manager review if applicable	_____	_____
3. Preparation of PR	_____	_____
4. Supervisory review	_____	_____
5. Accounting effort related to initiation, commitment and obligation of funds	_____	_____
6. Establishment and maintenance of due-in records	_____	_____
7. Internal control of PR	_____	_____
8. Technical coordination associated with PR preparation. (Does not include cost of maintaining technical data files, but does include cost of adding technical data to the PR whether accomplished manually or by automated process.) May include:	_____	_____
a. Cataloging and standardization review	(____) (____)	(____) (____)
b. Determination of quality control provisions to be inserted in contract	(____) (____)	(____) (____)
c. Technical decisions concerning source (competitive versus non-competitive) and engineering data requirements	(____) (____)	(____) (____)
d. Packing and preservation review	(____) (____)	(____) (____)

- |  |        |        |
|--|--------|--------|
| e. Provisioning data screening           | \$ ( ) | \$ ( ) |
| f. Legal review                          | ( )    | ( )    |
| g. Transportation data review            | ( )    | ( )    |
| h. Review of technical handbook adequacy | ( )    | ( )    |

B. Purchase

Either subparagraphs 1 or 2 below will apply for the "purchase" function, depending on whether the value is below or above \$2,500.

- |   |              |            |
|---|--------------|------------|
| 1. For small purchase items   | <u>Labor</u> | <u>ADP</u> |
| a. Receipt and recording of PR  | \$ _____     | \$ _____   |
| b. Solicitation effort  | _____        | _____      |
| (1) PR review   | ( )          | ( )        |
| (2) Determination of method of procurement  | ( )          | ( )        |
| (3) Obtain source list  | ( )          | ( )        |
| (4) Draft and type solicitation   | ( )          | ( )        |
| (5) Accomplish solicitation   | ( )          | ( )        |
| c. Evaluation and award effort  | _____        | _____      |
| (1) Price/cost analysis   | ( )          | ( )        |
| (2) Selection of contractor   | ( )          | ( )        |
| (3) Draft and type contract   | ( )          | ( )        |
| (4) Purchase office review  | ( )          | ( )        |
| (5) Legal review  | ( )          | ( )        |
| (6) Distribution of contract  | ( )          | ( )        |
| 2. For all other items  |              |            |
| (For call-type contracts, include only those functions relating to the processing of orders.) |              |            |
| a. Receipt and recording of PR and assignment of buyer  | _____        | _____      |

- |   |       |       |       |       |       |   |
|---|-------|-------|-------|-------|-------|---|
| b. Solicitation effort  | \$    | _____ | \$    | _____ |       |   |
| (1) Procurement planning  | (     | _____ | )     | (     | _____ | ) |
| (2) PR review and small business coordination                       | (     | _____ | )     | (     | _____ | ) |
| (3) Determination & finding   | (     | _____ | )     | (     | _____ | ) |
| (4) Determination of type contract                                  | (     | _____ | )     | (     | _____ | ) |
| (5) Synopsis and/or preliminary invitation notice                   | (     | _____ | )     | (     | _____ | ) |
| (6) Draft and type solicitation                                     | (     | _____ | )     | (     | _____ | ) |
| (7) Accomplish solicitation   | (     | _____ | )     | (     | _____ | ) |
| c. Evaluation and award effort                                      | _____ |       | _____ |       |       |   |
| (1) Receive quotes & proposals                                      | (     | _____ | )     | (     | _____ | ) |
| (2) Opening of bids   | (     | _____ | )     | (     | _____ | ) |
| (3) Evaluation (technical, procurement, production, transportation) | (     | _____ | )     | (     | _____ | ) |
| (4) Selection of probable contractor                                | (     | _____ | )     | (     | _____ | ) |
| (5) Selection of contractor   | (     | _____ | )     | (     | _____ | ) |
| (6) Procurement/legal review  | (     | _____ | )     | (     | _____ | ) |
| (7) Draft and type contract   | (     | _____ | )     | (     | _____ | ) |
| (8) Process administrative commitment document                      | (     | _____ | )     | (     | _____ | ) |
| (9) Forwarding of contract to contractor for signature              | (     | _____ | )     | (     | _____ | ) |
| (10) Receipt of contract and final review, signature                | (     | _____ | )     | (     | _____ | ) |
| (11) Obligation of funds  | (     | _____ | )     | (     | _____ | ) |
| (12) Distribution of contract and final administrative procedures   | (     | _____ | )     | (     | _____ | ) |

B. <u>Receipt and Payment</u>	<u>Labor</u>	<u>ADP</u>
1. Unload and check-in of material received	\$ _____	\$ _____
2. Quality inspection	_____	_____
3. Matching receipt papers	_____	_____
4. Relocation of materiel during receipt processing	_____	_____
5. Movement of materiel to warehouse	_____	_____
6. Updating storage location and asset records	_____	_____
7. Updating ICP asset records	_____	_____
8. Processing DD 250 and invoices for payment	_____	_____
9. Other financial effort related to payment	_____	_____

II. DIRECT LABOR/ADP COST PER ITEM ADMINISTERED AT A DEFENSE CONTRACT ADMINISTRATION SERVICES REGION (DCASR)

Note These costs will be determined by Defense Contract Administration Services (DCAS) and Defense Contract Audit Agency (DCAA) and published by OASD(I&L) for use by all Military Departments and the Defense Supply Agency.

A. Initial File Establishment	\$ _____	\$ _____
B. Pre-award Survey	_____	_____
C. Price/Cost Analyses	_____	_____
D. Production Follow-up	_____	_____

III. LABOR BENEFIT COSTS (See DODI 7041.3)

A. Personnel Benefits (health insurance, retirement, life insurance, disability) will be computed at 8% of direct labor cost.	\$ _____
B. Leave Entitlements to Cover Sick and Annual Leave, Holiday Leave, Administrative leave will be computed at 21% of direct labor cost.	\$ _____

IV. INDIRECT LABOR/SUPPORT COSTS NOT INCLUDED IN I AND II	<u>Total \$</u>
A. Communication Costs (Autodin, Telephone, Teletype)	_____
B. Internal Reproduction Equipment Rental	_____
C. Cost of Printing PRs and Contracts	_____
D. Materiel and Supplies	_____
E. Cost of Mail	_____
F. Data Service (Key Punch, Sort, the Variable Automatic Data Processing Costs Associated with Each Function)	_____
G. Personnel Support (Civilian Personnel Office)	_____
V. TOTAL VARIABLE COST TO ORDER	
Sum of Direct Labor/ADP Cost at ICP	_____
Sum of Direct Labor/ADP Cost at DCASAR	_____
Sum of Labor Benefit Cost	_____
Sum of Indirect Labor/Support Costs	_____
TOTAL	=====

COMPUTER PROGRAM TO EVALUATE CURRENT MODEL

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0036 GC TC 13
0037 7.1-5.1-4.48*P
0038 13 R=INT(OL+2*1.25*XMAD)
0039 GC TC 44
0040 10 M=111.25*XMAD**2/OL
0041 M=APA*(M,1.01)
0042 M=M-1
0043 M=OL/*1
0044 PR(1,1)=EXP(1-1*2*ALGQ(M))
0045 IF (PR(1,1) .GT. 6.91G) TO 40
0046 PR(1,2)=PR(1,1)
0047 NC=2
0048 42 PR(1,1)=(M2*FLOAT(NU-2)/FLOAT(NU-1))*(M1/M)*PR(NU-1,1)
0049 PR(1,2)=PR(1,1)*PR(1,2)
0050 PF=1-PR(1,2)
0051 IF (PF .LE. .81G) TO 41
0052 NC=NC+1
0053 GC TC 42
0054 41 R=NC-1
0055 GC TC 44
0056 73 R=1
0057 C DETERMINE REORDER QUANTITY AND COMPUTE COST OF SAFETY STOCK
0058 44 Q=SQRT((8.*D1*AI)/(1.*C))
0059 T=12.*C1
0060 Q=MAX(QW,1-D1)
0061 IF (Q .GT. INT(T)) Q=INT(T)
0062 S1=K*STOCK*Q*FLOAT(R)
0063 IF (S1 .NE. 0.1G) TO 18
0064 IF (CNCR(1,1) .EQ. 3.1G) TO 53
0065 IF (CNCR(1,2) .GT. FLOAT(J)) TO 50
0066 GC TC 15
0067 50 TVC=TV*BO*100.*1.*C*(CNHAND+ONHAND)/2.
0068 IF (J .NE. 8.1G) TO 100
0069 GC TC 55
0070 C DETERMINE TIME REQUISITION ARRIVED, ITS RELATIONSHIP TO ANY ORDER,
0071 C AND UPDATING RECEIVED VS FILLED AND INV POSITION
0072 18 TIMEP=J-1*(J)
0073 IF (CNCR(1,1) .LE. 0.1G) TO 14
0074 IF (TIMEP .GT. CNCR(1,2)) TO 15
0075 14 IF (J) .GT. CNHAND TO 16
0076 ONPAC=CNPAC-C(J)
0077 X=X+C(J)
0078 Y=Y+C(J)
0079 GC TC 17
0080 X=X+C(J)
0081 Y=Y+C(J)
0082 16 Y=Y+P*ANU
0083 PC=PC+C(J)-CNHAND
0084 ONPAC=C
0085 17 IF (CNHAND+CNUEIN=BO)
0086 22 IF
0087 C COMPARE IF WITH R, REORDER QUANTITY THAT IS NECESSARY AND DETERMINE
0088 C COST
0089 IF (IF .GT. FLCAT(R)) TVC=TV*BO*100.*1.*C*(CNHAND+ONHAND)/2.
0090 IF (IF .GT. FLOAT(R)) TO 49
0091 IF (C .EQ. 0.1G) PAX(180,1.)
0092 N=N+1
0093 CNCR(N,2)=TIMEP+XLT
0094 32 OLEIN=CNUEIN+C
0095 CNCR(N,1)=CNCR(N,1)+Q
0096 CCST=CCST+C
0097 22=22+FLOAT(Q)
0098 IF (22 .GT. FLCAT(R)) TO 33
0099 TVC=TV*BO*100.*1.*C*(CNHAND+ONHAND)/2.
0100 GC TC 56
0101 C STATEMENT 1-95 UPDATE INCRDER QUEUE
0102 15 IF (N .GT. 1) TO 19
0103 ONPAC=CNHAND+CNCR(1,1)
0104 GC TC 20
0105 19 SLP=CNCR(1,1)
0106 CC 21 *2,N
0107 IF (TIMEP .GT. CNCR(N,2)) TO 22
0108 SLP=SLP+CNCR(N,1)
0109 GC TC 21

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0123 22 GC TC 23
0124 23 CCATTJUE
0125 23 ONPANC=CNHANC+SUM
0126 23 DLEIN=OLEIN-SUM
0127 23 IFIONPAND .GE. 80100 TO 25
0128 23 BC=BC-CNANC
0129 23 ONHANC=C.
0130 23 GC TO 24
0131 23 UNPANC=CNHANC-EL
0132 23 BC=0.
0133 26 N1=1
0134 26 IF(CACRDRIN,2) .LT. TIMREC100 TO 250
0135 26 GC 24 P=K,N
0136 26 CACRDR(P,1)=JNCRDRIN,1)
0137 26 CACRDR(P,2)=CNCRDRIN,2)
0138 26 N1=N1+1
0139 24 CCATTJUE
0140 24 N=N+1
0141 24 N=N+1
0142 24 GC TC 251
0143 250 N1
0144 251 N=1
0145 251 CCATTJUE
0146 251 GC 27 N1=N,E
0147 251 GC 28 N2=1,2
0148 251 ONCRRR(P1,M2)=0.
0149 28 CCATTJUE
0150 27 CCATTJUE
0151 27 IF(C1) .EQ. 0.100 TO 50
0152 27 GC TC 14
0153 20 ONCRRR(1,1)=0.
0154 20 CACRDR(1,2)=0.
0155 20 DLEIN=C.
0156 20 IFIONPAND .GE. 80100 TO 35
0157 20 BC=BC-CNANC
0158 20 ONPANC=0.
0159 20 N=N+1
0160 20 IF(D1) .EQ. 0.100 TO 50
0161 20 GC TC 14
0162 35 ONPANC=CNHANC-BC
0163 35 GC=C.
0164 35 N=N+1
0165 35 IF(D1) .EQ. 0.100 TO 50
0166 35 GC TC 14
0167 94 IF(J) .LT. 8100 TO 100
0168 94 AVGSTK=STOCK/8.
0169 97 XX=XX+X
0170 97 YY=YY+Y
0171 97 XPC=X/X
0172 97 WRITE(6,202)K3,X,Y,XMCE,AVGSTK,COST
0173 202 FORMAT(1X,16.8X,F8.2,11X,F8.2,8X,F6.4,7X,F10.2,5X,F10.2)
0174 97 K3=K3+1
0175 97 TCTCCS=TCTCCS+CCST
0176 97 TCTSTK=TCTSTK+AVGSTK
0177 100 CCATTJUE
0178 100 GC TC 101
0179 102 WRITE(6,103)
0180 203 FORMAT(1X,10X,'TCTAL REORDER COST',10X,'TOTAL STOCK COST',10X,'XMCE',
0181 103 '10X',10X)
0182 103 XPC=X/Y
0183 204 WRITE(6,204)TCTCCS,TCTSTK,XMOE,TVC
0184 204 FORMAT(16X,F10.2,17X,F10.2,12X,F6.4,8X,F10.2)
0185 204 STCP
0186 204 ERLG SUBCHK
0187 204 ENC

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## COMPUTER PROGRAM TO EVALUATE ALTERNATIVE MODEL

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 K11=K11B1,2,3,4,5,6

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ITEM	REQUESTED	FILLED	NOE	STOCK	CCST
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TOTAL REORDER COST  
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TOTAL STOCK COST  
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## LIST OF REFERENCES

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